Optimisation of Ground Stations Location in Aeronautical Multilateration Systems

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***Abstract*— The main goal of this thesis is to analyse the performance of the current multilateration systems installed in Portugal, by NAV Portugal, and to establish a set of recommendations for the future multilateration systems implementations. These goals were accomplished by the development and implementation of a set of models and algorithms: difference between WAM and ADS-B routes, analysis and dimensioning algorithm and selection algorithm. In order to achieve the intended analysis several simulations on different scenarios were performed, which were the Lisbon, Azores and Porto airports. From the analysis of the simulation results, it is possible to conclude that the WAM systems installed in the Lisbon and Azores regions perform according to the parameters set by the system´s manufactures, that is, for example when analysing the difference between WAM and ADS-B routes, for the Lisbon scenario under a 30 NM radius, the error is under 100 m and for the Azores scenario under a 100 NM radius the error is under 300 m. And finally, that the airplane´s position error decreases as the airplane navigates towards the ground stations.**

Keywords*— Surveillance; Multilateration; WAM System; ADS-B System; Ground Stations.

I. INTRODUCTION

Since the moment that Humankind put the first airplanes in the air the necessity of a system that could ensure the security of everything and everyone involved in this huge operation was created. From this situation arose the concept of monitoring the big areas (air-ground area) where airplanes circulate, the initial surveillance system came from this concept.

Surveillance systems are necessary for air traffic control, being then possible to detect and send information about the airplane's position, identification, and altitude. Initially, these systems were mainly composed of primary and secondary radars, but now, with new technologies available, such as multilateration (MLAT), there are new and highly reliable systems, although complex and expensive [1].

These technologies have been used through the years and have their vantages and advantages. The Primary Surveillance Radar (PSR) is a radar that works with an echo, which simply detects objects, in our case airplanes, without any particular specificity. In terms of energy, these radars have very high levels of consumption, and there is a possibility that the received signal can be lost. The main goal of PSR systems is to ensure the airplane's landing and taking-off. These systems can only detect and position the airplane. Like any other system, the PSR has advantages and limitations. On the one hand, there is not one single object in the air space that can be invisible to the 'eyes' of air traffic controllers, and in addition no other equipment is necessary, hence, it is only needed one site per installation and the infrastructure costs are low. On the other hand, it has its cons, since it cannot provide the airplane's identification, and because it uses an echo, it has a limited range and can only work in Line of Sight (LoS), so no installation in mountainous areas is possible.

Then in order to overcome the limitations of PSR, regarding costs, reliability and performance, the Secondary Surveillance Radar (SSR) was created. With the SSR it is possible to exchange information between the airplanes and the ground station in large surveillance areas, and to detect the position of a particular airplane (altitude and identification). The SSR system is composed by radar that operates on the ground and a transponder that goes on-board the airplane. The communication between radar (on the ground) and the transponder (at the airplane) works based on queries and replies that are coded. The radar interrogates the airplane's transponder at 1 030 MHz, which triggers the transponder on-board to reply at 1 090 MHz with the airplane's identification and altitude. It is important to refer that the airplane's position is not known by the same interrogation/reply process. This last information is obtained by the airplane itself by calculating the turnaround time from the radar to the airplane, in other words, by measuring the time difference between the interrogation and the transponder reply message. By knowing the airplane´s position, identification, and altitude, it is possible for the air traffic control system to have the airplane's position.

Although not fully implemented (i.e. implementation expected between 2020 and 2025) the Automatic Dependent Surveillance (ADS) will be the surveillance system for the near future. In ADS, unlike the previous discussed methods, there is no need for interrogations and replies, because the airplane itself can determinate its position using the navigation system on-board. ADS is a satellite-based technology, and there are two different modes of using this type of surveillance, ADS-Contract (ADS-C) and ADS-Broadcast (ADS-B). ADS-C works by using the airplane´s navigation system and determines its position, velocity, and meteorological data, but as the name indicates, it works by contract. This technology is used in areas (e.g., mountainous or oceanic areas) where the use of radar is not possible, because it has no range. But it is the ADS-B that is used in complement to MLAT systems; in this case, the airplane has to have a Global Positioning System (GPS) receiver on-board, since it is used to obtain the airplane's position.

Nowadays, in order to be a relevant surveillance system, it is necessary to meet a set of requirements. The system has to provide an estimation of the position, altitude of the airplane, and identify it. An air-ground surveillance system is characterised by coverage volume, accuracy, integrity, update rate, reliability and availability [2]. And, according to Eurocontrol [3], Air Navigation Service Providers (ANSPs) should choose their surveillance systems based on operational requirements, cost benefit assessment, and safety assessment. The MLAT system is accurate, efficient, cheap (comparing with "common" radars), safe, and has a large coverage. With these set of characteristics, MLAT is nowadays the surveillance system that is being used and implemented all over the world.

Although SSR brings many advantages, it cannot provide ground surveillance and the requirements of latency and update rate need to be improved. Improvements have been done, and currently one uses Multilateration systems. The MLAT systems do everything that the SSR does with a plus, one can know the exact location of an airplane.

Multilateration systems enable the location of an airplane based on the TDOA (Time Difference of Arrival) method, which are the main focus of study in this thesis. In order to provide and calculate the airplane's location, ground stations (i.e., sensors) are spread throughout airport´s areas to enable total air traffic surveillance. The main goal of this thesis is to achieve the optimal number and location of ground stations, in order to supply the angular and spatial resolution required to obtain the airplane's location. The work in this thesis aims to be relevant on future implementations of multilateration systems in Portugal.

The idea of a safer and more reliable air surveillance system is behind the motivation for this thesis. The present work is focused on assessing the performance of the MLAT systems installed by NAV in Portugal. In order to achieve the goal of this thesis, several steps were taken. An assessment of the multilateration systems installed by NAV was done, implying a study of the basic aspects of MLAT systems, an analysis of the current systems performance, an optimisation of the ground stations' location, and recommendations for future implementations. These final recommendations result in a proposal that translates into the set of ground stations to be used in order to implement a WAM (Wide Area Multilateration) system and generate a minimum error associated with the airplane's position.

The work presented in this thesis is the result of a direct collaboration with NAV Portugal E.P.E., which resulted in providing all the information necessary to accomplish the goals of this thesis, and a close follow up of its progress.

This paper is composed of 5 sections, including the present one. In Section II, the technical principals of a multilateration system will be described.

In Section III, the theoretical equations that are the basis of the TDOA algorithm are described, and the models and algorithms that were developed are presented. Algorithms were developed to calculate the difference between WAM and ADS-B routes, to load all the information needed to build the hyperbolas, and to build the hyperbolas themselves. All the models resulted in the implementation of a simulator.

In Section IX, the different scenarios under analysis are described, and the simulator output results are presented as well as their analysis. To conclude, recommendations for a future WAM installation are made.

The final Section of the thesis will briefly resume every conclusion drawn from the work but also gives a more global analysis of the problem under study. This Section presented a brief introduction to the problem under study and the main work will be shown in the next Sections.

II. Multilateration

MLAT is basically a set of sensors that are displaced in a certain way, in order to obtain an airplane's position and identification. This information is generated by the signals that are produced by the transponders and by the use of TDOA techniques. This way, one can track airplanes in a very accurate form.

MLAT provides surveillance for modes A/C, mode S and ADS-B. There are two types of MLAT systems, LAM (local area multilateration) and WAM, which basically differ on the sensors coverage area. LAM is more appropriate to airplanes and vehicles surveillance at the airport area, while WAM has a wide area system, i.e., the sensors are widely spread in order to ensure the coverage area. They also differ in the number of antennas necessary to install and their location, which are consequences of the difference in the size of the coverage areas. Since the SSR has a wide coverage area, the most suitable choice to replace this one is the WAM system.

MLAT is based on TDOA, which analyses the airplane's received signal and the sensors' received signals. A different number of sensors lead to different accuracy results. The number of sensors cannot be less than 3, because with 3 sensors one has the object's two-dimensional location, i.e., the target would have to be on the ground. In order to have the target's three-dimensional location, one needs 4 or more sensors, enabling to know where the target is in the air. But with 4 sensors, one can only determine the location of one airplane at the same time (which in real life is unpractical, since one needs to know the position of various airplanes at the same time, and this makes the problem a very complex one), and it also creates a number of problems, such as synchronisation of the received times and precision of processed times. Hence, in real life, the number of sensors has to be larger than 4 in order to have a reliable system, a larger precision and the total coverage of the intended area.

III. THEORETICAL MODELS

In order to achieve the goal of this thesis there were performed two different types of analysis: one in which the difference between WAM and ADS-B routes is calculated, from now on called route difference analysis, and another in which the maximum error that an airplane's position can take is calculated, designated by error position analysis.

The error position analysis is applied in two different scenarios: First, in which all the information related to the used ground stations and the airplane routes from WAM and ADS-B systems are available and second, in which the only information available is the ADS-B route, hence, being an analysis of the set of ground stations that can be used to minimise the location error.

A. Theoretical Equations

As explained in Section II, the MLAT system is based on the TDOA algorithm, enabling to determine the position of an airplane accurately. In order to do this, it is necessary to measure the time of arrival (TOA) of the signals exchanged between the ground stations and the airplane. This technique can also be named hyperbolic positioning, because it is based on the intersection of the hyperbolas that are the direct result of the TDOA algorithm. Each hyperbola corresponds to the time difference of arrival between the signal transmitted by the airplane and received by one of the ground stations. After having all the hyperbolas, their intersection will provide the precise location of the airplane at that particular time.

The distance between the ground station and the airplane is calculated by [4]:

$$D_{i[m]} = \sqrt{(x_{[m]} - x_{i[m]})^2 + (y_{[m]} - y_{i[m]})^2 + (z_{[m]} - z_{i[m]})^2},$$

$$i \in \{1, 2,3,4 ..., n_{gs}\} \quad (1)$$

where:

- (x_i,y_i,z_i): Location of the ith ground station.
- (x,y,z) : Position of the airplane.
- n_{gs} : Number of ground stations.

In order to achieve the hyperbolas intersection and therefore to determine the position of the airplane, it is necessary to have an equation for each of the hyperbolas [4]:

$$\sqrt{(x_{[m]} - x_{i[m]})^2 + (y_{[m]} - y_{i[m]})^2 + (z_{[m]} - z_{i[m]})^2}$$
$$-\sqrt{(x_{[m]} - x_{1[m]})^2 + (y_{[m]} - y_{1[m]})^2 + (z_{[m]} - z_{1[m]})^2} =$$
$$D_{i[m]} - D_{1[m]} = c_{[m/s]} \times (t_{i[s]} - t_{1[s]}),\ i \in \{2,3,4...,n_{gs}\} \quad (2)$$

where:

- c: Speed of light.
- t : Time when the airplane sent a signal.
- t_i : Time when the sensor received the signal.

The perfect scenario consists of an ideal situation, in which there is an error equal to zero associated with the hyperbolas formation. According to [5], there are errors that occur throughout the multilateration process that can be identified and quantified. These errors have different sources associated with them. The errors can be considered random and systematic, being divided into timing errors, propagation errors, surveying errors and reference errors.

So adapting these findings to (2) the following results are achieved [4]:

$$\sqrt{(x_{[m]} - x_{i[m]})^2 + (y_{[m]} - y_{i[m]})^2 + (z_{[m]} - z_{i[m]})^2}$$
$$-\sqrt{(x_{[m]} - x_{1[m]})^2 + (y_{[m]} - y_{1[m]})^2 + (z_{[m]} - z_{1[m]})^2} + n_{i,1[m]} =$$
$$D_{i[m]} - D_{1[m]} + n_{i,1[m]} = c_{[m/s]} \times (t_{i[s]} - t_{1[s]}) + n_{i,1[m]},$$
$$i \in \{2,3,4...,n_{gs}\} \quad (3)$$

where:

- $n_{i,1}$: Quantified error.

The difference between WAM and ADS-B routes is accomplished by having all the readings from the airplane's positions using the WAM system. And at the same time, the GPS system installed in the airplane is also collecting data for each position that the airplane is taking.

Both these readings result on the route of the airplane, Figure 1, but with some differences, and these differences can be analysed by calculating the distance between the WAM route and the ADS-B one, Figure 1. This task is accomplished by calculating the distance between a point and a line, (4), the point being at the WAM route and the line at the ADS-B one, [6]:

$$d_{[m]} = \frac{\left|(y_{n+1} - y_n) \times x_1 + (x_{n+1} - x_n) \times y_1 + x_{n+1} \times y_n - y_{n+1} \times x_n\right|}{\left|\sqrt{(y_{n+1} - y_n)^2 + (x_{n+1} - x_n)^2}\right|} \quad (4)$$

where:

- x_1 and y_1 are the coordinates of point, P.
- x_n, y_n, x_{n+1} and y_{n+1} are the coordinates of points, Q_n and Q_{n+1}, on the line.
- d corresponds to the distance between the point and the line.

Figure 1 shows that in fact the WAM and ADS-B routes do not overlap.

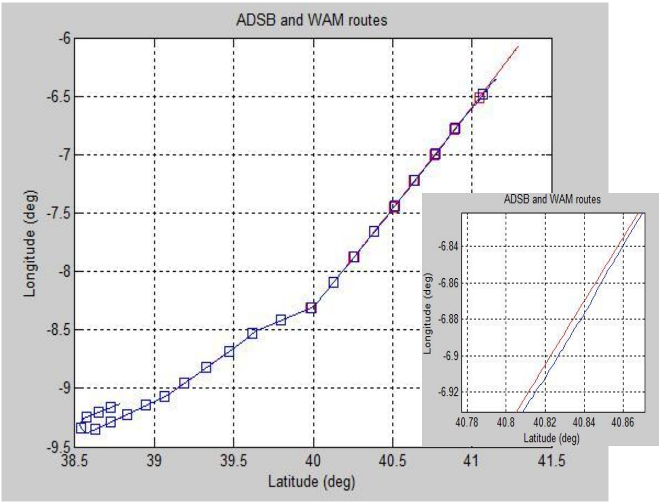

Figure 1 – Difference between WAM (blue) and ADS-B (red) routes.

B. Difference Between WAM and ADS-B routes

The flowchart for the analysis of the difference between WAM and ADS-B routes is shown in Figure 2. WAM and ADS-B routes must be selected, and from the WAM route information one knows the initial airplane's position, P_{targ}. Then, a search on the ADS-B route information is performed, in order to find the coordinates that are above and below the airplane position, Q_m and Q_{m+1}, respectively. After collecting this information, it becomes a distance between point and line problem, P_{targ} being the point, and Q_m and Q_{m+1} the two points defining the line.

Finally, after computing this distance problem using (4), one obtains the expected result, the difference between both routes. One should note that this result is useful when evaluating the influence that the airplane's approximation to the ground stations has on the system.

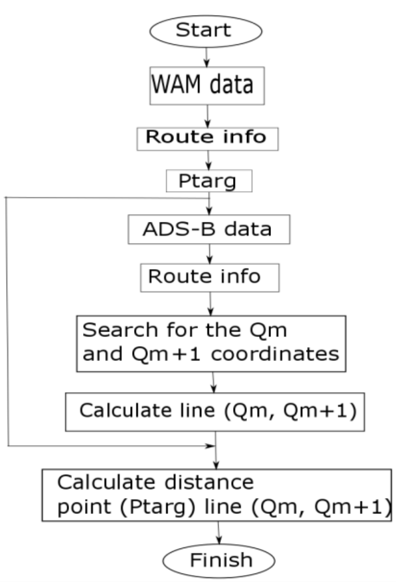

Figure 2. Difference between WAM and ADS-B routes.

C. Analysis and System Dimensioning Algorithm

In order to determinate the maximum error of an airplane's position, two fundamental steps to achieve this goal are the analysis and system dimensioning, in Figure 3 and Figure 4, respectively.

Figure 3 shows the flowchart of the steps necessary to load all the information needed to build the hyperbolas. First, the WAM route is loaded and the information related to the route and the set of ground stations that is used for each route position is extracted. Then, a similar process is done to the ADS-B data. ADS-B data are more reliable, because this information is given by the airplane´s GPS, the difference in between WAM and ADS-B routes being calculated in order to obtain the coordinates of the point that is the new target position. All the collected information is used to calculate the distance between the airplane and every ground station, and the distances in between all ground stations using (1). These results are used to create the hyperbolas that allow the determination of the maximum error that an airplane's position can take.

In order to complete the process of error determination, it is necessary to build the hyperbolas, Figure 4: for each pair of different ground stations a hyperbola is created, and then an error to the hyperbolas is applied. This error generates two situations: one in which the error is equal to zero and then all the hyperbolas intersect in one point, that point being the airplane's position, implying that the multilateration system is well implemented in the simulator; another in which the error assumes a non-zero value, and then the different pairs of hyperbolas associated with the error intersect and create an area, and from the points that form this area the one that is the farthest from the airplane is considered to be the maximum error that the airplane's position takes.

One should note that, in Figure 4, the S variable represents the set of ground stations that is used in each position of the route and the N one represents the number of ground stations.

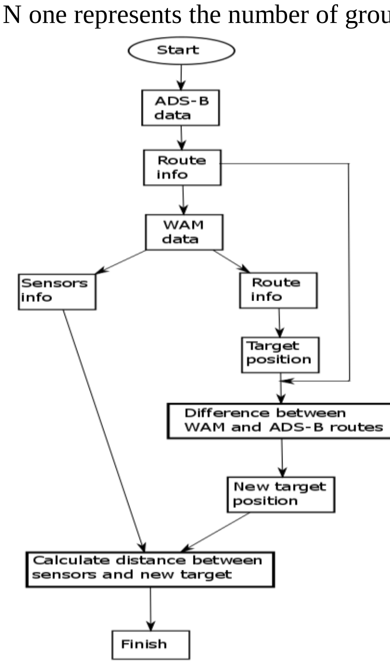

Figure 3. Analysis – information loading.

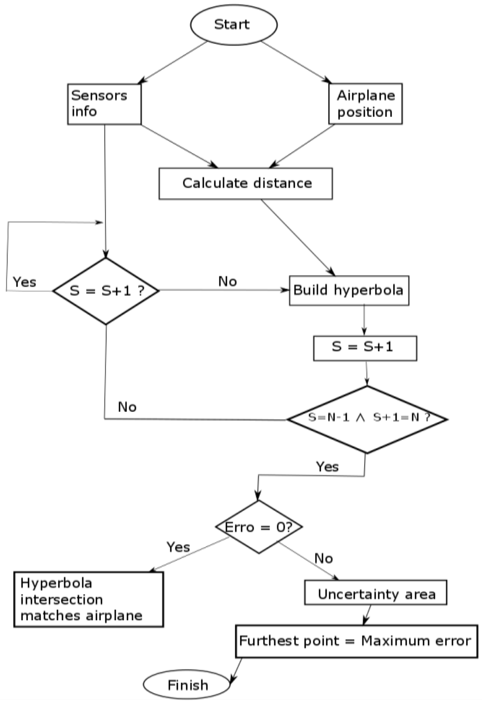

Figure 4. System dimensioning – hyperbola building.

D. Selection Algorithm

The selection algorithm is used in situations where the WAM route information is not available, and therefore the set of ground stations that are used for each airplane's position is not known. In these situations, the ADS-B route data are the only available information, Figure 5.

First, an ADS-B route must be selected and loaded, where all ground stations positions must be known and loaded. After collecting this crucial information, it becomes a combination problem. For each airplane's position, the distance between all ground stations and the airplane is calculated using (1), then the *m* ground stations that are closer to the airplane are the ones selected to be taken into the combination problem.

Combinatorial analysis was chosen to find all the different sets of ground stations that can provide a minimum error associated with a given airplane's position, combinations being [7]:

$$C_k^m = \binom{m}{k} = \frac{m!}{k!(m-k)!} \qquad (5)$$

where:

- *m* represents the total of ground stations that will be taken.
- *k* represents the number of ground stations that the combination will have, this value varying between a minimum and a maximum, the latter being *m*.

The result of the combinatorial analysis is a set of subsets of *k* distinct elements of *m*. Each subset represents a possible combination of ground stations, therefore, for each of the subsets the system dimensioning will be applied. Finally, the error associated with each of the ground stations combination is obtained, and then the one that presents the minimum value is the combination of ground stations that should be used on the MLAT system for that particular airplane's position.

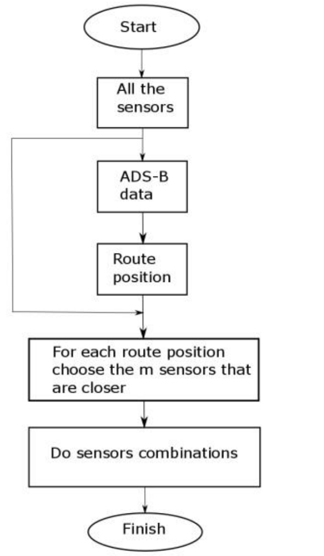

Figure 5. Selection of the ground stations.

This section describes the implementation of the models and algorithms that were developed to be used in a simulator. This simulator uses Matlab2013b and was developed to enable the analysis of the different scenarios presented in Section IV. The main structure of the simulator is presented in Figure 6, and although there are different scenarios being simulated, implying that different algorithms are used, the general structure of the simulator does not change.

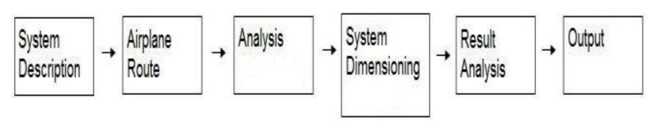

Figure 6. Main structure of the simulator.

IV. RESULT ANALYSIS

In this section, the different scenarios to be analysed are presented, as well as the different flight routes that are considered in the simulations.

A. Scenario Description

1) Lisbon scenario

The maximum error calculation associated with the WAM system at the Lisbon airport fits the situation in which all the information related to the used ground stations and the flight routes from WAM and ADS-B systems are available, as explained in Section III.

The WAM system in Lisbon is composed of 8 ground stations, Figure 7. The flight routes used for the simulation are shown in Figure 7, for both WAM and ADS-B routes.

2) Azores Scenario

The Azores region scenario analysis is done under the same conditions as the Lisbon one, i.e., information regarding all ground stations and the WAM and ADS-B routes are available, Figure 8.

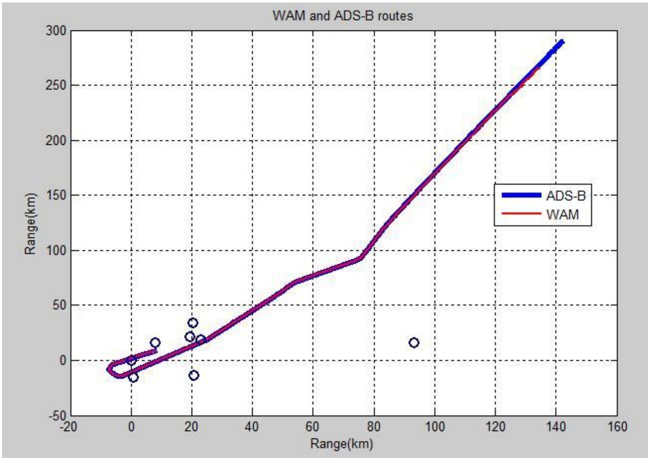

Figure 7. WAM and ADS-B routes for Lisbon scenario.

The WAM system of the Azores region is composed of 17 ground stations, their positions being presented in Figure 8.

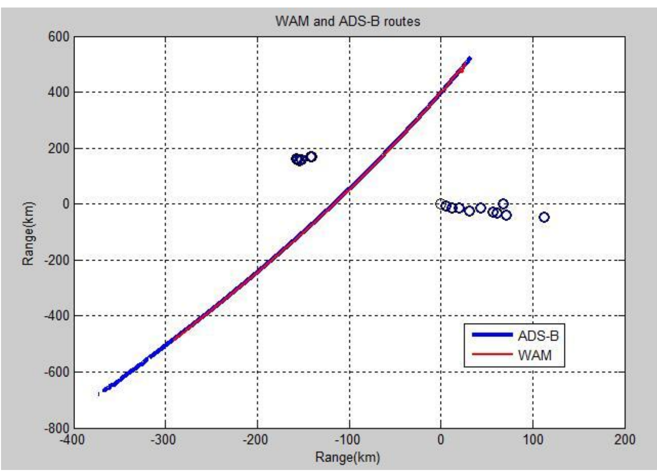

Figure 8. WAM and ADS-B routes for Azores scenario.

3) *Porto Scenario*

The Porto region scenario fits the situation, described in Section III, in which the only information available is the ADS-B flight route, Figure 9, and the ground stations position. Because the WAM system is not fully operational, the goal for the Porto region scenario analysis was to determine the set of ground stations that provide the minimum system error, this information being useful to building the actual WAM system under deployment.

The Porto scenario is composed of 12 ground stations, their positions being presented in Figure 9. Although there are 12 ground stations, two of them, RU07 and RU10, have the same location so for simulation purposes only one of them is considered.

B. Diffence Between WAM and ADS-B routes analysis

This section presents the results of the Lisbon and Azores regions simulations. One presents the difference between the WAM and ADS-B routes, hence, being possible to verify that as an airplane gets closer to the ground stations the difference between the routes decreases.

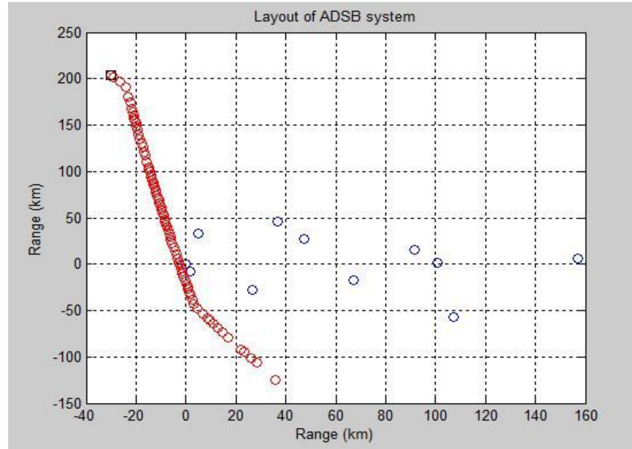

Figure 9. Porto ADS-B route and ground stations.

In the Lisbon scenario, one knows that the MLAT system has a range of 30 NM (i.e. 55.6 km), meaning that the system is designed to ensure airspace surveillance under a 30 NM radius, with the centre at the Lisbon airport.

The results for the Lisbon region, Figure 10, show that the difference between routes decreases when the airplane is closer to the ground stations. One should note that around the 55 km (30 NM) red mark system requirements are satisfied, since the error is mostly less than 100 m.

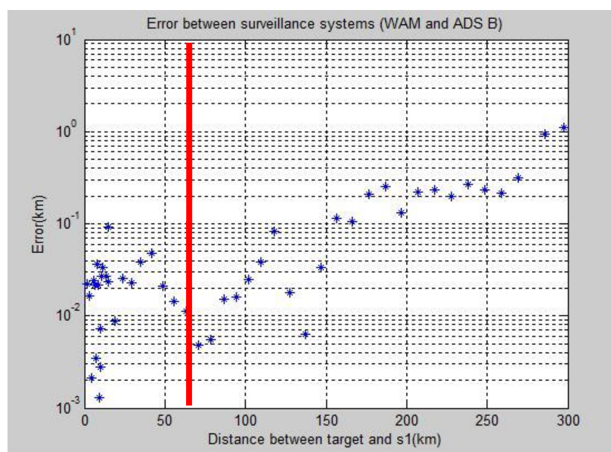

Figure 10. Difference between WAM and ADS-B routes for Lisbon.

For the Azores region, a similar analysis was performed, but with different requirements, since the two MLAT systems are not equal. Figure 11 presents the simulations results for Azores, and, as expected, the conclusions are the same, i.e., as the distance between the airplane and the reference ground station, the ground stations with coordinates (0,0), gets smaller, the difference in between the WAM and ADS-B routes also gets smaller. The reference value for the error in the Azores scenario is around 300 m, and in the area in which the airplane is closest to the reference ground station, this value is accomplished, since around that area the error is mostly less than 300 m, showing that the implemented system is meeting the requirements and technical specifications set for it.

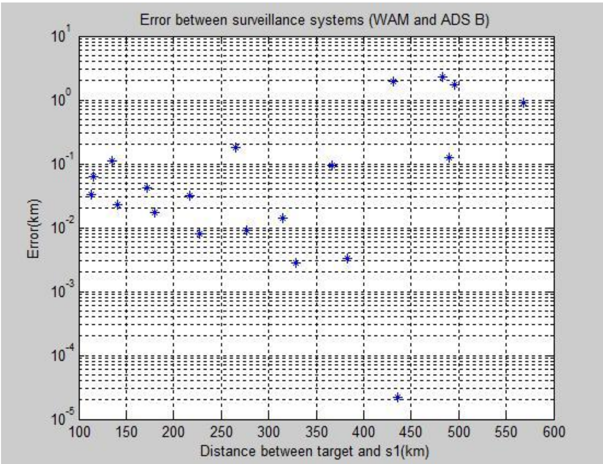

Figure 11. Difference between WAM and ADS-B routes for Azores.

A statistics analysis was made in order to better comprehend the data that resulted from the Lisbon route difference analysis.

The samples that were introduced in the *Curve Fitting Toolbox*, are presented in Figure 12. To compute the fitting distribution analysis, the Exponential distribution was chosen. The results for the goodness-of-fit parameters for the analysis of the difference between WAM and ADS-B routes results, for Lisbon region, are presented in Table I.

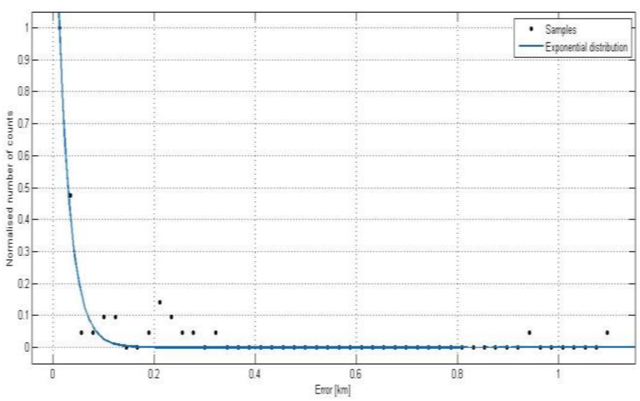

Figure 12 – Normalised number of measurements fitted with an exponential distribution for the Lisbon scenario.

Table I - Goodness-of-fit parameters.

Region	Lisbon
Distribution	Exponential
R-square	0.9381
Adjusted R-square	0.9368
RMSE [km]	0.0391

It is valid to say that the R-square and the adjusted R-square present values close to 1 and a RMSE value very close to 0. These results mean that the fitting is appropriate to the set of samples. The fitting model applied to the Lisbon scenario explains 93.8% of the total variation in the data, and since the RMSE value is so close to 0, this indicates that the fit is useful for prediction.

C. Set of Ground Stations

From the results presented in this section, it is possible to observe the variation of the number of ground stations that belong to the set selected to determine an airplane's position.

After analysing the Lisbon route and the number ground stations used throughout the flight route, the results are presented in Figure 13.

From the results presented in Figure 13, it is possible to observe that for this Lisbon route, when the airplane navigates at distances above 100 km from the reference point, the number of used ground stations varies in between 4 and 6, in the majority of the cases. When this distance decreases to the interval [25, 100] km, the number of ground stations varies mostly in between 6 and 8. Finally, when the airplane is approaching the reference ground station, the number of ground stations varies mostly, from 4 to 8.

Since the Lisbon system is designed to perform on a 30 NM radius, under this 55 km radius it is possible to observe, Figure 13, that as the airplane navigates in the interval [25, 55] km, the number of ground stations varies between 6 and 8 confirming that more ground stations are necessary to achieve higher performance values. And as the airplane approaches the reference ground station, and therefore gets closer to the airport ground, the number of ground stations needed to perform the TDOA algorithm decreases confirming that when an airplane circulates near the ground less ground stations are needed for the system to perform. This confirms that the simulations results are validated by the theoretical results.

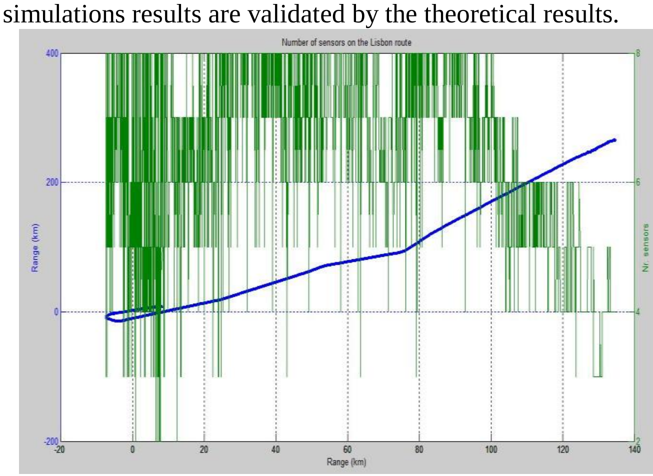

Figure 13 – Number ground stations used throughout Lisbon route.

Since the information regarding the set of ground stations used in the MLAT process is provided by the WAM route file, the same type of analysis was performed for the Azores scenario, the results being presented in Figure 14: from left to right, one can see that in the interval [250, 300] km, the interval in which the airplane is the farthest from the reference ground station, the number of ground stations varies from 4 to 5: in [200, 200] km, this number values between 5 and 10; when the airplane gets closer to the ground stations, in [50, 200] km, the number of ground stations increases to vary between 10 and 15: and finally, towards the end of the flight route, in [0, 50] km, the number of ground stations decreases to vary from 5 to 10. In conclusion, when the airplane navigates towards the reference ground station, the number of ground stations used increases.

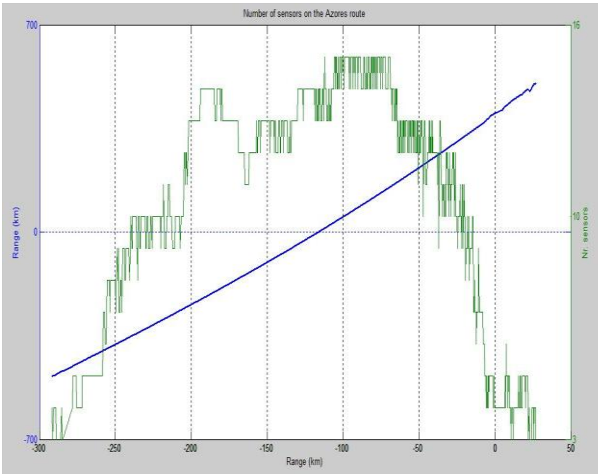

Figure 1 - Number ground stations used throughout Azores route.

After analysing the results for the Lisbon and Azores regions, the same conclusions are reached. As the distance between the airplane position and the respective reference ground station decreases, the number of used ground stations increases, and when the airplane reaches the minimum distance, the maximum number of used ground stations is reached.

D. Error Analysis

This section is dedicated to the analysis of the maximum error achieved when calculating an airplane position. In this section, one presents results for Lisbon and Azores. In order to correctly determine and analyse the evolution of the error throughout the considered routes, it is necessary to know the set of ground stations used in each route position, Figure 13 and Figure 14.

One of the first stages of the simulation is to know the flight route that has been loaded to execute the simulation, all the ground stations that belong to given scenario, the airplane position that has been taken under analysis, and the set of ground stations used to perform the TDOA algorithm for that airplane's position.

From the set of ground stations information, the hyperbolas are formed, but depending on the error value applied to the hyperbolas, two different outcomes are possible: on the one hand, if the error is equal to zero, all hyperbolas should intersect in one point, which corresponds to the airplane's position, validating that the simulator is well developed, Figure 15; on the other hand, if the error is not equal to zero, the hyperbolas intersection results in an area, Figure 16.

In Figure 16, it is possible to determine the uncertainty area and, consequently, the point of this area that is the farthest from the airplane's position, which is the one considered to be the maximum error that the airplane's position can reach. In order to evaluate the variation of the maximum error throughout Lisbon's and Azores' routes, measurements were made.

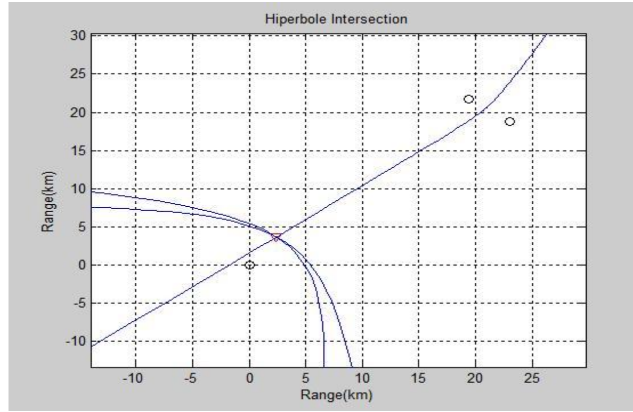

Figure 15 – Hyperbolas intersection with error equal to zero.

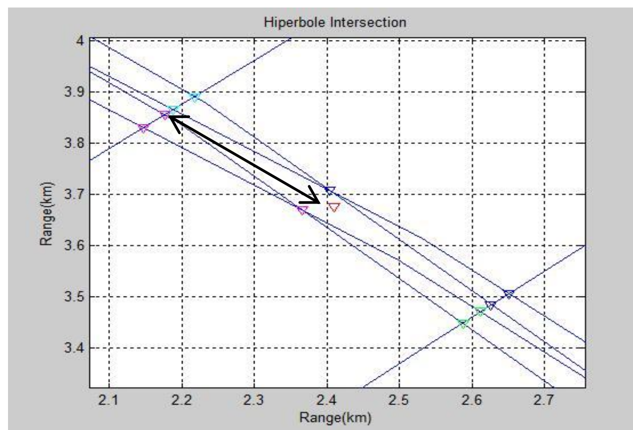

Figure 16 – Uncertainty area.

In [5] it is possible to find all the errors that contribute to the MLAT process in detail, their sources and numerical contributions. The summation of all the errors make up the final value of 52 m [5], but following a NAV's recommendation, the value considered in the simulator was half of the original, meaning that for the calculations regarding the error from this point forward, the value of the error is 25 m. This adjustment happens because the true value used by the MLAT system manufacturer is not available, so corrections were made in order to achieve the results known from the actual system.

The error decreases as the airplane gets closer to the ground stations. As the area formed by the boundaries, i.e., hyperbolas that result from the error introduction gets smaller, the results for the maximum error decrease.

In both scenarios, Lisbon and Azores, it is possible to verify, Figure 17 and Figure 18, respectively, that the error position results follow the same trend. As the distance between airplane and reference ground station get smaller, the values of the airplane's position error also decrease. Corresponding the minimum error value to the minimum distance between airplane and reference ground station.

In figure 17 it is possible to observe that in the interval between [0, 50] km, the results for the error associated to the airplane's position suffers a slight elevation. This results from the curvature that the Lisbon's route takes in this interval, Figure 7.

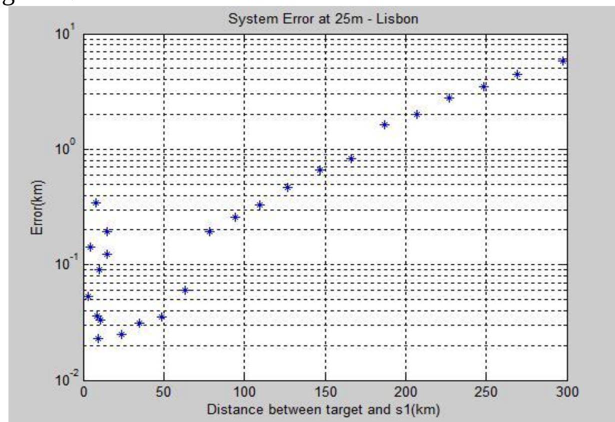

Figure 17 – Lisbon simulation results with error at 25 m.

In Figure 18 it is clear to observe the decrease of error's value as the airplane navigates towards the ground stations. Assuming at the minimum distance between airplane and reference ground station, the error values are less than 100 m.

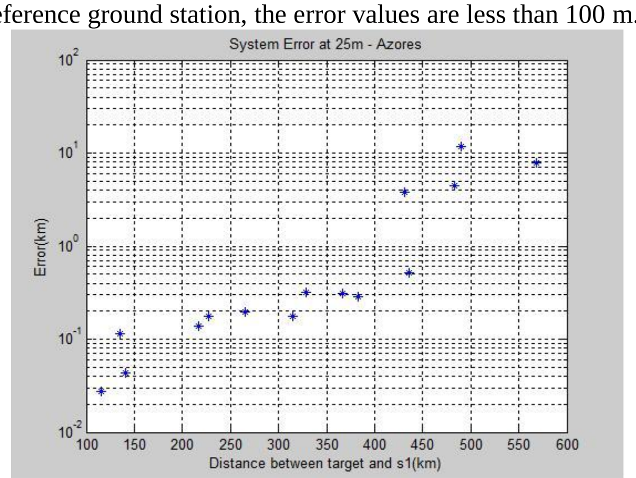

Figure 18- Azores simulation results with error at 25 m.

E. Porto Analysis

Since the WAM system for the Porto region is not fully implemented, the goal of this section is to determine the set of ground stations that provide the best result for each of the airplane's position in the route under analysis, as explained in Section III.

Following a NAV recommendation for the Porto region analysis, for each airplane's position in the ADS-B route, the distance between all ground stations and the airplane's position was calculated via (1), and the 6 ground stations that are closer to the airplane's position were selected. Finally, from this 6 ground stations, the set that provides the best solution was chosen.

In order to achieve this goal, a set of different combinations using (5) was taken, resulting in a series of subsets of k distinct elements of the selected 6 ground stations, assuming a value from 4 to 6, resulting in a total of 22 possible combinations.

For each of the 22 possible ground stations combinations, the maximum error that an airplane's position can achieve was calculated. Finally, the combination that presents the minimum error value is considered to be the set of ground stations to be recommended to be used for the WAM system for that particular airplane's position.

In Figure 20, it is possible to observe the results achieved for the minimum error and in Figure 19 the number of ground stations that provides that optimum result. As expected, as the distance between the airplane and the ground stations gets smaller, the error associated with the airplane's position decreases.

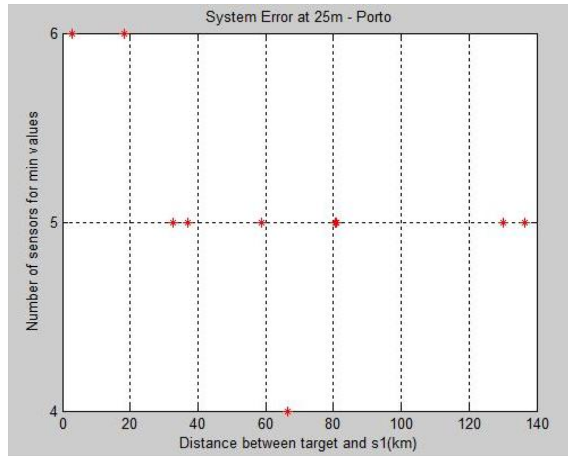

Figure 19 – Number of ground stations that provide the minimum error.

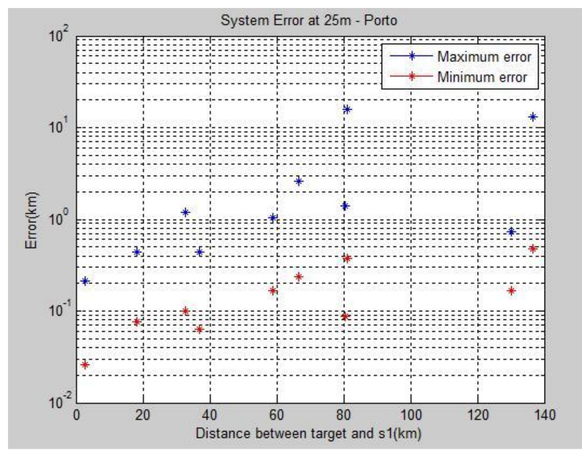

Figure 20 – Comparison of results regarding the maximum and minimum error associated with each airplane's position.

From Figure 20, it is possible to perform the comparison of results between the minimum and the maximum error associated with an airplane's position when analysing the Porto route provided for simulation. A similar analysis was done in order to obtain the number of ground stations that provide the maximum error. It is possible to verify that this results always in a combination of 4 ground stations, because, as expected, the number of intersecting hyperbolas increases the uncertainty area, so if the goal is to find the maximum error, it is crucial to find the largest areas, therefore, the area formed by the smallest number of hyperbolas and, consequently, the smallest number of ground stations.

V. Conclusions

The main goal of this thesis was the study of multilateration systems, focusing on the assessment of the performance of the systems installed in Portugal, in Lisbon and Azores, and some design recommendations for the Porto WAM system. The developed models and algorithms are used to analyse two different situations. One in which the difference between WAM and ADS-B routes is calculated, and another in which the maximum error that an airplane's position can take is calculated. This second analysis is applied in two different scenarios, one in which all the information related to the used ground stations and the airplane's routes from WAM and ADS-B systems are available, and another in which the only information available is the airplane ADS-B route, and because of that, the analysis is a dimensional one of the set of ground stations that is used in order to minimise the system's error.

Through the developed algorithms, i.e., the difference between WAM and ADS-B routes algorithm, analysis and system dimensioning algorithms, and selection algorithm, it was possible to implement the described models in a logical way. Having all the necessary pieces to assemble the simulator, the simulator was developed and prepared to provide the output results for each of the different scenarios and situations to be analysed. From this simulator, it is possible to obtain results for the maximum error of an airplane's position throughout the flight route and the differences between WAM and ADS-B routes. A more detailed analysis of each conclusion follows: from the difference between WAM and ADS-B routes and the Lisbon and Azores scenarios analysis, it is possible to achieve the same set of conclusions; as the airplane gets closer to the ground stations, the difference between the routes decreases. After using the *Curve Fitting Toolbox*, it is safe to say that the Exponential distribution provides a good fitting, since for Lisbon the R-square is equal to 0.938 and for Azores it is 0.900.

Regarding the set of ground stations analysis for the Lisbon and the Azores scenarios, it is possible to conclude that the number of ground stations used for the implementation of the TDOA algorithm varies throughout the flight route, but this variation follows the same trend: the number of ground stations increases as the distance between the airplane and the reference ground station deceases. When evaluating the results of the maximum error that an airplane can achieve for Lisbon and Azores, the conclusions are the same. In both scenarios, the value of the error added to the system is equal to 25 m, and under these conditions the obtained results are satisfactory. The error associated with Lisbon decreases as the airplane navigates towards the reference ground station, and when analysing the area under the 30 NM radius, the error presents values under 100 m. A similar analysis for the Azores leads to the same conclusions, but under a different radius area, this time a 100 NM radius, the error presents values under 300 m. In both cases, the results obtained from the simulator meet the margins recommended by NAV.

The results of the analysis for Porto are different, because the goal was to determine the set of ground stations that provide the best result for each of the airplane's positions in the route under analysis. It is possible to conclude that the number of ground stations has a direct influence on the size of the uncertainty area, since as the number of ground stations increases the size of the uncertainty area decreases. As expected, as the distance between the airplane and the reference ground station decreases the error value also decreases.

Regarding the values of the error, 25 m, used for simulation purposes, it is not that accurate, since it was not possible to obtain the real value used by the manufacturer of the MLAT system used by NAV, rather being obtained through a trial and error process.

After analysing all the results, it is possible to validate some of the initial hypotheses: the bigger the number of ground stations, the wider the range of the system and the smaller the error associated with it, the terrain typology, the building surrounding the airports, the mountainous areas are all factors that influence system's performance and add to its error. Is necessary to use more ground stations as the airplane gets closer to the reference ground station, due to the spatial resolution problem. As the distance gets smaller, the 'opening' of the angle of the ground stations also gets smaller creating the necessity to use more ground stations.

Acknowledgment

To the engineers from NAV Portugal, namely Eng. Carlos Alves, Eng. Luís Pissarro and Eng. André Maia, and to Prof. Luís M. Correia, whose contribution for the presented work was fundamental. The knowledge shared and the input and feedback given played a crucial part at this thesis' developing.

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